

NAG Toolbox for MATLAB

f07ge

1 Purpose

f07ge solves a real symmetric positive-definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by f07gd, using packed storage.

2 Syntax

```
[b, info] = f07ge(uplo, ap, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07ge is used to solve a real symmetric positive-definite system of linear equations $AX = B$, the function must be preceded by a call to f07gd which computes the Cholesky factorization of A , using packed storage. The solution X is computed by forward and backward substitution.

If **uplo** = 'U', $A = U^T U$, where U is upper triangular; the solution X is computed by solving $U^T Y = B$ and then $UX = Y$.

If **uplo** = 'L', $A = LL^T$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^T X = Y$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates how A has been factorized.

uplo = 'U'

$A = U^T U$, where U is upper triangular.

uplo = 'L'

$A = LL^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The Cholesky factor of A stored in packed form, as returned by f07gd.

3: **b**(ldb,*) – double array

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, \text{nrhs_p})$

The n by r right-hand side matrix B .

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The first dimension of the array **ap** and the second dimension of the array **ap**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix A .

Constraint: $n \geq 0$.

2: **nrhs_p** – **int32 scalar**

Default: The second dimension of the array **b**.

r , the number of right-hand sides.

Constraint: **nrhs_p** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters

1: **b(ldb,*)** – **double array**

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, \text{nrhs_p})$

The n by r solution matrix X .

2: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs_p**, 4: **ap**, 5: **b**, 6: **ldb**, 7: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** = 'U', $|E| \leq c(n)\epsilon|U^T||U|$;

if **uplo** = 'L', $|E| \leq c(n)\epsilon|L||L^T|$,

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \operatorname{cond}(A, x) \epsilon$$

where $\operatorname{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty} \leq \operatorname{cond}(A) = \| |A^{-1}| |A| \|_{\infty} \leq \kappa_{\infty}(A)$.

Note that $\operatorname{cond}(A, x)$ can be much smaller than $\operatorname{cond}(A)$.

Forward and backward error bounds can be computed by calling f07gh, and an estimate for $\kappa_{\infty}(A)$ ($= \kappa_1(A)$) can be obtained by calling f07gg.

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to f07gh to refine the solution and return an error estimate.

The complex analogue of this function is f07gs.

9 Example

```
uplo = 'L';
ap = [2.039607805437114;
      -1.529705854077835;
      0.2745625891934577;
      -0.04902903378454601;
      1.640121946685673;
      -0.2499814119483738;
      0.6737303907389101;
      0.7887488055748053;
      0.6616575633742563;
      0.5346894269298685];
b = [8.699999999999999, 8.300000000000001;
     -13.35, 2.13;
     1.89, 1.61;
     -4.14, 5];
[bOut, info] = f07ge(uplo, ap, b)

bOut =
    1.0000    4.0000
   -1.0000    3.0000
    2.0000    2.0000
   -3.0000    1.0000
info =
      0
```